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Vortex Flow Model for the Blade-Vortex Interaction Problem

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Introduction

IN the present paper, a highly efficient vortex flow model is developed to simulate the attached two-dimensional blade-vortex interaction (wherein boundary layers remain attached throughout the interaction process). Discrete vortex dynamics have been previously employed by other researchers in studying inviscid incompressible flows.[1,2] The present work differs from the previous studies in three major points. 1) The present work utilizes a viscous theory of aerodynamics to evaluate the aerodynamic load. 2) The work combines a theoretical solution for the boundary vortex sheet with a numerical procedure for the wake vortices. 3) In the present analysis, an accurate procedure is established for the simulation of the vortex shedding process, which is crucial to the accuracy of solutions of all inviscid unsteady flows.

Recent experimental observations by Poling and Telionis[3] indicate that the present trailing-edge flow model is physically realistic. The results from this model tend to the steady-flow solution asymptotically as the change in circulation vanishes, and the model allows vorticity to be shed from both sides of the trailing edge. In the present model, the location and the strength of the nascent vortex are determined separately. The usual difficulty that the strength of the nascent vortex is oversensitive to its location is thus eliminated.

Closed-form solutions for the unsteady lift, drag, and moment experienced by an airfoil encountering a vortex passing near it are obtained by applying a general theorem of aerodynamics.[4] These closed-form solutions allow the evaluation of vortex-induced unsteady loads without tedious calculations. Furthermore, these solutions allow the contributions of the various vortical regions coexisting in the interaction flowfield, such as the wake and the boundary layer, to be evaluated individually. The relative importance of each vortical region can then be assessed. Because of the simplicity of these formulations, they can be easily extended to the study of the noise induced by blade-vortex interaction.[5] Selected results obtained by the present vortex flow model are presented and compared with other numerical solutions in this paper.

Presented as Paper 86-1094 at the AIAA/ASME 4th Fluid Mechanics, Plasma Dynamics, and Lasers Conference, Atlanta, GA, May 12–14, 1986; received July 1, 1986; revision received Jan. 4, 1988.

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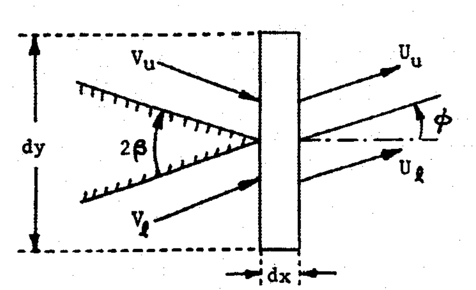
Vortex Flow Model for Blade-Vortex Interaction

In high-Reynolds number external flows, the vorticity is generally concentrated in regions near the solid body. Under some circumstances, simplified models can be utilized in analyzing the flow. For an attached blade-vortex interaction problem, the vortical region is composed of three distinct components: the passing vortex, i.e., the oncoming vortex that interacts with the blade; the boundary layer at the blade surface; and the wake trailing the blade. An accurate solution of the flowfield depends on the correct modeling of these three vortical components.

In the present study, the passing vortex Ω is represented by a point vortex, which is an accurate representation provided that the vortex is outside of the boundary layer during the interaction. In high-Reynolds number flows, the boundary layer is very thin compared to the characteristic length of the solid body. The vorticity in the boundary layer can be represented by a vortex sheet γ. Using ω to denote the vorticity field, the vortex sheet strength can be defined as $\gamma = \int_0^\delta \omega \mathrm{d}n$, where δ is the boundary-layer thickness. The wake is a continuation of the boundary layer. That is, the vorticity in the boundary layer is continuously transported downstream and shed into the wake. In the present study, the wake vorticity is represented by a series of point vortices, Γ_i, $i = 1,2,\ldots,N$, successively shed from the airfoil's trailing edge. Both the passing vortex and the wake vortices are allowed to convect with the local fluid velocities v_i, which are determined by the Biot-Savart law. Except for the nascent vortex, the positions of vortices are determined at each step by simple numerical integration.

Nascent Wake Vortex

In order to determine the strength and the position of a nascent vortex, three flow quantities at the airfoil's trailing edge are needed. These are 1) the vorticity shedding rate, 2) the flow velocity, and 3) the flow direction. At the trailing edge of the airfoil, in the absence of flow separation, the boundary layers on the two sides of the airfoil merge into a single wake layer. Since in the present model the boundary layers of finite thickness are represented by vortex sheets of zero thickness, the smoothly turning streamlines at the outer edges of the boundary layers are replaced by two streamlines that have finite turning angles at the trailing edge. In an unsteady real fluid flow, the wake vortex sheet is in general not parallel to either the upper or the lower surface of the airfoil trailing edge. The direction of the wake layer can be determined by applying the momentum theorem to an infinitesimal control volume at the airfoil trailing edge, as shown in the sketch.



For a sufficiently small control volume, the momentum equation can be written as

$$\int_{cs} \rho v(v \cdot \mathrm{d}s) = 0 \tag{1}$$

where v is the fluid velocity and cs is the control surface. The given vector expression can be written as two scalar equations:

$$(V_u^2 + V_\ell^2)\cos^2\beta = -(U_u^2 + U_\ell^2)\cos^2\phi \tag{2}$$

$$(V_u^2 + V_\ell^2)\cos\beta\sin\beta = (U_u^2 + U_\ell^2)\cos\phi\sin\phi \tag{3}$$

where the velocities V_u, V_ℓ, U_u, U_ℓ, and the angles β and ϕ are as defined in the sketch. Dividing Eq. (3) by Eq. (2) leads to

$$\tan\phi = [(V_\ell^2 - V_u^2)/(V_\ell^2 + V_u^2)]\tan\beta \tag{4}$$

The velocities U_u and U_ℓ can be obtained by applying the continuity equation to the control volume:

$$U_u = V_u(\cos\beta/\cos\phi) \tag{5}$$

$$U_\ell = V_\ell(\cos\beta/\cos\phi) \tag{6}$$

The vorticity shedding rate and thus the strength of a nascent vortex, as will be shown, can be determined by the velocity values given in Eqs. (5) and (6). Since a vortex sheet strength can be expressed as the difference of the tangential velocities across the vortex sheet, one has

$$\gamma_u = -V_u \tag{7}$$

$$\gamma_\ell = V_\ell \tag{8}$$

$$\gamma_w = U_\ell - U_u \tag{9}$$

where γ_u, γ_ℓ, and γ_w are the vortex sheet strengths of the upper boundary layer, the lower boundary layer, and the wake layer near the trailing edge, respectively. The flux of vorticity in the wake layer is obtained by integrating the product of the tangential velocity and the vorticity across the layer. With the fact that the vorticity magnitude is equal to the normal derivative of the tangential velocity in a thin shear layer, this integral yields $\frac{1}{2}(U_\ell^2 - U_u^2)$ as the flux of wake vorticity. This flux can also be written as $U_w\gamma_w$ where U_w is the velocity of the wake vortex sheet. Using these two expressions together with Eq. (9), one obtains

$$U_w = \tfrac{1}{2}(U_\ell + U_u) \tag{10}$$

The strength of a nascent vortex shed during a time interval Δt can be written as the vorticity flux times Δt:

$$\Gamma_k = \tfrac{1}{2}(U_\ell^2 - U_u^2)\,\Delta t \tag{11}$$

where the subscript k denotes the time step and Γ_k denotes the strength of the kth wake vortex.

Equations (4–11) permit the strength as well as the position of a nascent vortex to be determined using known values of γ_u and γ_ℓ. It is noted that the vorticity flux in the near wake given by Eq. (11) is slightly different from the combined total flux of vorticity in the two boundary layers. This difference can be explained as the effect of the finite turning angles of the streamlines at the trailing edge of the airfoil.

It is important to point out that the presented trailing-edge flow model is established on the basis of an observation of the viscous flow near the trailing edge. It is not, and cannot be, obtained on the basis of potential flow analysis. A viscous flow solution near the trailing edge shows that in the absence of massive flow separation, the vorticity is shed from both sides of the trailing edge. It should be emphasized, even at the risk of appearing trivial, that potential flow analyses are useful only in the absence of massive flow separation. While the Giesing-Maskell[2] model does not reflect the fact that vorticity is shed from both sides of the trailing edge, the present model simulates this phenomenon correctly. It is interesting, though, that Eq. (4) reduces to the Giesing-Maskell assumption if the vorticity is shed from one side of the trailing edge only. The fact that a solution from the present model would approach steady state continuously without a sudden change in flow direction at the trailing edge also makes the model a more realistic representation of real fluid flows.

Boundary Vortex Sheet Strength

In the present study, the airfoil is mapped onto a circle of radius R using conformal mapping. The boundary vortex sheet strength can be evaluated analytically in the circle plane (r,θ) as[6]

$$\gamma(\theta) = -2U_\infty(\sin\theta + \sin\alpha) + \frac{\Omega}{2\pi}\frac{R - r_o\cos(\theta_o - \theta)}{R^2 + r_o^2 - 2Rr_o\cos(\theta_o - \theta)} - \sum_{k=1}^{N}\frac{\Gamma_k}{2\pi R}\frac{r_k^2 - R^2}{R^2 + r_k^2 - 2Rr_k\cos(\theta - \theta_k)} \tag{12}$$

where θ_k and r_k give the location of the kth wake vortex and θ_o and r_o give the location of the passing vortex. It can be shown,[6] by integrating γ to give the bound circulation around the airfoil, that the law of total vorticity conservation is satisfied by Eq. (12).

Vortex-Induced Unsteady Force

An aerodynamic theorem that relates the force and moment experienced by an immersed solid body to the vorticity field around the body was developed by Wu.[4] For the present blade-vortex interaction problem, this theorem can be simplified as[6]

$$F = F_x + iF_y = i\rho\frac{\mathrm{d}}{\mathrm{d}t}\int_{R_\infty} z\omega\,\mathrm{d}R \tag{13}$$

where F_x and F_y denote the drag and the lift, $z = x + iy$ is the complex coordinate in the physical plane, and ω is the vorticity distribution. This theorem can be applied separately to the previously mentioned three distinct vortical regions in the interaction flowfield to give unsteady force components associated with each of the vortical systems present in the flow. Using F_1 to denote the force component associated with the wake, F_2 the component associated with the boundary layer, and F_3 the component associated with the movement of the passing vortex, Eq. (13) gives[6]

$$F_1 = i\rho\sum_{k=1}^{N} V_k\Gamma_k \tag{14}$$

$$F_2 = i\rho\Omega\left[V_o\left|\frac{\mathrm{d}\zeta}{\mathrm{d}z}\right|_o^2 + R^2\frac{\overline{V_o}}{\bar{\zeta}_o}\left|\frac{\mathrm{d}\zeta}{\mathrm{d}z}\right|_o^2 - V_o\right] + i\rho\sum_{k=1}^{N}\Gamma_k\left[V_k\left|\frac{\mathrm{d}\zeta}{\mathrm{d}z}\right|_k^2 + R^2\frac{\overline{V_k}}{\bar{\zeta}_k}\left|\frac{\mathrm{d}\zeta}{\mathrm{d}z}\right|_k^2 - V_k\right] \tag{15}$$

$$F_3 = i\rho\Omega(V_o - U_\infty) \tag{16}$$

where $V_o = u_o + iv_o$ is the complex velocity of the passing vortex, $V_k = u_k + iv_k$ is the velocity of a wake vortex, and z and ζ denote the physical and the transformed plane, respectively.

Results and Discussion

With the flow model and the closed-form solutions for the unsteady force, the two-dimensional blade-vortex interaction can be studied without extensive numerical calculation. A systematical study of the blade-vortex interaction has been carried out by the present authors using this and other theoretical and numerical models. Selected results for the case of a vortex of strength $\Omega = -0.2U_\infty c$ passing a NACA 0012 airfoil are presented here. A series of the positions of the wake vortices during the interaction are given in Fig. 1. When the passing vortex is far away from the airfoil, weak vortices are shed from the airfoil trailing edge. The trajectory of these weak vortices is nearly aligned with the bisector of the trailing edge, as is also observed in viscous flow solutions.[6] If one used the Giesing-Maskell model, then the wake vortex trajectory would have to be always aligned with one side of the trailing

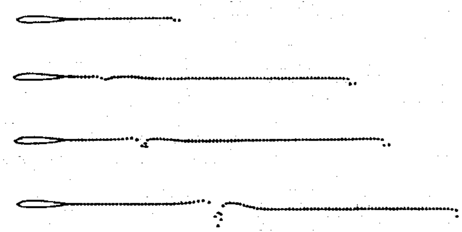


Fig. 1 Vortical wake predicted by vortical flow analysis.

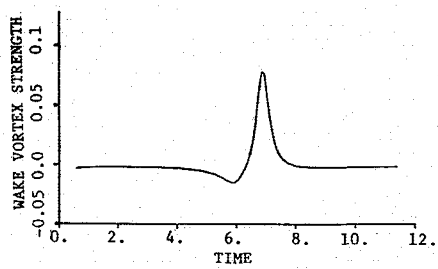


Fig. 2 Time history of vortex shedding.

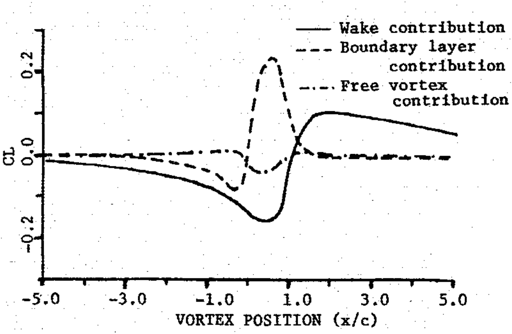


Fig. 3 Unsteady lift components predicted by vortical flow analysis.

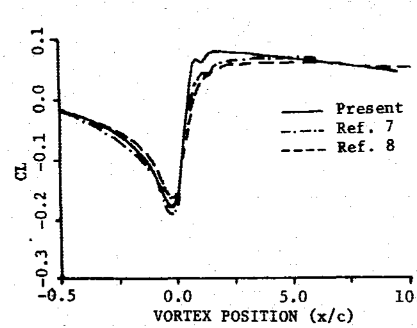


Fig. 4 Unsteady lift compared with transonic flow solution.

edge, and the flow picture would be distorted. Figure 2 shows the strengths of the wake vortices plotted against the time. In Fig. 3, the unsteady lift components induced by the three vortical regions are plotted separately. These curves show that the most important factor in this blade-vortex interaction is the vortex shedding process. Figure 4 shows the total vortex-induced unsteady lift as compared to the viscous flow solutions given by McCroskey and Goorjian[7] and by Sankar and Tang.[8] The good agreement between the present results and those of others from viscous flow solvers indicates that the present vortex flow model simulates high-Reynolds number attached unsteady flows very accurately.

Conclusion

A vortex flow model for the problem of two-dimensional blade-vortex interaction has been established, and closed-form solutions for the vortex-induced unsteady force are derived. A new trailing-edge flow model (unsteady Kutta condition) is introduced; this trailing-edge flow model is physically more meaningful than the conventional stagnation point method. It has been established that the present approach can accurately predict the vortex-induced unsteady force without tedious computations.

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Recirculation Structure of the Coannular Swirling Jets in a Combustor

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Introduction

SWIRL is used extensively in gas turbine combustors to achieve high-combustion efficiency, stable combustion over a wide range of operation, good temperature transverse quantity, and minimum size. In the study of swirling flows, the term "vortex breakdown" refers to the formation of a recirculation zone on the axis of flow.[1] The swirl-induced recirculation zone is an important phenomenon of combustor research.

Previous experiments and computations[2–4] have focused upon single or multiple swirling jets generated by swirl tubes in combustors. Recently, So et al.[5] observed that the swirling flowfield generated by vane swirler possesses two recirculation regions in the center of the test model, one very close to the swirler hub and one further downstream. They could not explain the reason why there are two recirculations coexisting. Due to the fact that the first recirculation is very small compared to the second, and the axial velocity between these two

Presented in part as Paper 87-0305 at the AIAA 25th Aerospace Science Meeting, Reno, NV, Jan. 12–15, 1987; received May 8, 1987; revision received Nov. 23, 1987.

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